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THE THEORY OF THE VISCOSITY OF HELIUM-II:
I. COLLISIONS OF ELEMENTARY EXCITATIONS IN HELIUM-II

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[A Digest]

The final results of this study may be summarized as follows:

1. The nonzero matrix elements of the Fourier components of density equal the following:

$$(d_p)_n, n+1 = (d_{0p}/2c) \cdot (n+1)^{\frac{1}{2}} \cdot \exp-i\omega t \text{ and}$$

$$(d_p)_n, n-1 = (d_{0p}/2c) \cdot n^{\frac{1}{2}} \cdot \exp-i\omega t.$$

Here d_p is the density of He-II in connection with momentum p ; n is the number of phonons of momentum p ; c is the sonic velocity in He-II; and ω is the cyclic frequency connected with the quantum relation $\omega = cp/\hbar$ (\hbar here is Planck's bar- h).

2. The effective cross section of scattering of phonon, by phonon, as calculated by the perturbation method, equals:

$$\sigma_{\text{ph-ph}}(p, p_1) = 6 \cdot 10^{-19} (\pi T)^4.$$

Here π is momentum expressed in units of $\hbar T/c$ so that $p_1 = \pi(\hbar T/c)$. T is the absolute temperature and k is Boltzmann's constant. Also, initial p is assumed to be much smaller than colliding p_1 .

3. The effective cross section of scattering of phonon by roton equals:

$$\sigma_{\text{ph-rot}}(\text{phonon, roton}) = 7 \cdot 10^{-19} (\pi T)^4.$$

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4. The effective cross section of scattering of roton by roton equals:

$$\sigma(r, r') = 5 \cdot 10^{-15} / T^2.$$

Here it was assumed that the mean velocity (\bar{v}) of the rotons is related to mass (m) thus: $p = m\bar{v} = (2/3.14) \cdot (kT/q)$.

The above four theoretical results were derived separately after following a mathematical course of reasoning, which is given below for each of the four in an abbreviated manner:

1. The energy spectrum of He-II is not known exactly, but it is known that the energy E is a linear function of momentum p near the origin and has a minimum at $p = p_0$ say. This reasoning is according to W. Keesom and McWood (Physica 5, 737, 1938) and L. Landau (J. of Phys. 11, 91, 1941).

Near the minimum, E^2 is expressed as a quadrinomial in rising powers of p^2 with four unknown coefficients, which are easily determined from E being linear in p near $p = 0$ and having a minimum near $p = p_0$, as just mentioned. Then the square root of E^2 is extracted and the first two terms only kept, which turn out to be p and p^3 except for expansion-coefficients.

The density and velocity of He-II are expressed as Fourier series and their commutator is formed and equated to (h/i) grad (delta-function). Then the commutative relations of the Fourier components are obtained, after expanding the delta-function in a complex Fourier series.

2. First, the Hamiltonian (energy) per unit volume of He-II is expressed in two terms, kinetic and potential. Then the Hamiltonian is expanded into its unperturbed and perturbed parts expressed in terms of density deviations d' and sonic velocity c and v .

The six intermediate states, I-VI, of phonon momenta are listed and the matrix element of transition is found. The energy differences of the initial and intermediate states are expressed.

Finally, the differential cross section for four-phonon scattering is set up in terms of the matrix element of transition and the delta-function of energies, multiplied by the differential phase-volume of momenta in an orthogonal xyz system.

The total cross section is too complicated to calculate. Therefore, it is assumed for simplicity that one of the momenta of the colliding phonons is quite small in comparison to the scattered phonons' momenta, which simplifies the matrix element of transition in the differential formula for cross section. The differential cross section is transformed into polar form and integrated over the phase volume of momentum and then averaged over all angles formed by momenta.

Finally, $\sigma(p, p_1)$ is expressed literally in terms of a dimensionless parameter u equal to d_0/c times the partial derivative of c^2 with respect to density d and momentum p_1 (colliding). The quantity u is calculated from Keesom's data for density versus pressure (Helium, Elsevier, 1942), and the phonon momentum p is subject to Bose's statistics, thus giving the final temperature relationship.

3. The Hamiltonian H of the phonon-roton system is expressed as the sum of the phonon energy, roton energy, and the interaction energy between phonon and roton. The roton is treated as particle in a phonon field.

The Hermitian operator is formed from the roton-vector momentum and the velocity of the medium (the density is assumed to fluctuate with small oscillations). The interaction energy and density are related by expanding the roton

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energy in a power series in the deviation of density from its equilibrium value and saving only second-order terms.

The Hamiltonian is now finally studied by the perturbation method. The various partial derivatives with respect to density, now in H_0 , are found from experimental data. The final Eulerian equation relates velocity v and density d ; the equation is manipulated to obtain v and d separately as time functions.

Next, the probability of transition of a roton of momentum P to a roton of momentum P' is determined, with the two intermediate states and energy differences. The matrix element of transition is formed and the differential cross section of scattering, phonon by roton, is set up and solved by integration over all angles of scatter.

Inserting numerical values for mass m , velocity c , density d_0 , momentum P_0 , and coefficients determined in 1, and considering the phonons and rotons as a mixture of ideal gases, one finally obtains the temperature function of cross section.

4. The exact roton-roton interaction is completely unknown, but the probability of scattering of roton by roton can be calculated by the perturbation method. The energy of roton-roton interaction is considered as the delta-function of the distance between the rotons. The probability of transition of rotons from the initial state with momenta P and P_1 to the final state with momenta P' and P'_1 is given by the perturbation theory as: the differential of probability equals the square of the matrix element of the interaction energy times the delta-function of energy times the differential phase-volume of scattered momenta.

The wave functions of the rotons is set up and gives the matrix element of transition. The differential probability now has a simplified form. Finally, the total effective cross section of scattering of roton by roton is found by integrating the differential probability over the phase-volume of scattered particles.

5. In a future publication the final results, giving cross sections of scattering, will be used to find the temperature function of viscosity, the ultimate purpose of this work.

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